

Nonperturbative enhancement of nonGaussian correlators in de Sitter space

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We compute the four-point correlation function of a light $O(N)$ scalar field in de Sitter space in the large- N limit. For superhorizon momentum modes, infrared effects strongly enhance the size of loop contributions. We find that in the deep infrared limit, the latter are of the same order as the tree-level one. The resulting nonGaussian correlator presents a singular momentum dependence.

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Quantum field theory in curved spaces is a topic of great interest with a long history [1]. The case of de Sitter (dS) space has attracted a lot of attention both because of its large degree of symmetry and because of its phenomenological relevance for the early inflationary era and for the current accelerated expansion of the universe. Specific phenomena such as gravitational redshift or particle creation imposes one to rethink much of what is known in Minkowski space, starting from the basic notions of particle and vacuum state, even for free fields [2]. At present, free gauge fields, such as the photon or the graviton, are still the subjects of debates [3].

Interacting fields can be studied by means of perturbation theory [4–8]. They pose practical and conceptual issues. An example is the transPlanckian problem [9], i.e., the question of the effective decoupling between infrared (IR) and ultraviolet physics, which underlies the very concept of QFT on dS. They also reveal novel specific features as compared to the flat space case. For instance, scalar fields of sufficiently large mass—in units of the expansion rate—are fundamentally unstable and can decay to themselves [10]. Light fields, which have no Minkowski analog, are also of great interest because of their phenomenological relevance, e.g., for inflationary cosmology. They exhibit strong semi-classical fluctuations for superhorizon modes and turn out to be essentially nonperturbative, even at weak coupling, due to large IR effects [5, 11]. In recent years, various methods inspired from flat space techniques have been developed to deal with IR issues in dS. Results are still rather scarce but the nonperturbative aspects of light scalar fields are being unravelled [12–20].

A typical example is the phenomenon of dynamical mass generation: a field with vanishing tree-level mass develops an effective mass due to its self-interactions [11]. This lifts the flat tree-level potential and regulates possible IR divergences. Incidentally, this results in nonanalytic coupling dependences of physical observables. A similar phenomenon has been demonstrated for an $O(N)$ scalar field in the large- N limit in the case where the tree-level potential shows spontaneous symmetry breaking [15]. Strong IR fluctuations restore the symmetry, as anticipated in [21], and lead to nonperturbatively en-

hanced loop contributions [19].

Immediate phenomenological implications of nontrivial field interactions in the inflationary universe are possible quantum corrections to standard inflationary observables [6, 22], or the possibility of nonGaussian features of primordial density fluctuations [23]. It has been pointed out in [12] that IR effects may lead to parametrically enhanced nonGaussianities at tree-level. In this letter, we show that loop corrections to nonGaussian correlators are also amplified by IR effects and may in fact be as large as the tree-level contribution. We consider the case of an $O(N)$ theory with quartic interactions and employ a nonperturbative large- N limit. Using the expressions for the field propagator and four-point vertex function recently obtained in Refs. [15, 19], we compute the equal time four-point correlation function for the phenomenologically relevant case of superhorizon modes, which we obtain in closed analytical form. We point out the interesting momentum structure of the loop contributions.

Consider the $O(N)$ -symmetric scalar field theory with classical action (a sum over $a = 1, \dots, N$ is implied)

$$\mathcal{S}[\varphi] = \int_x \left\{ \frac{1}{2} \varphi_a (\square - m_{\text{dS}}^2) \varphi_a - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right\}, \quad (1)$$

with the invariant measure $\int_x \equiv \int d^{d+1}x \sqrt{-g}$, on the expanding Poincaré patch of a $d+1$ -dimensional dS space. In terms of comoving spatial coordinates \mathbf{X} and conformal time $-\infty < \eta < 0$, the line-element reads (we choose the Hubble scale $H = 1$)

$$ds^2 = \eta^{-2} (-d\eta^2 + d\mathbf{X} \cdot d\mathbf{X}). \quad (2)$$

In Eq. (1), the mass term $m_{\text{dS}}^2 = m^2 + \xi \mathcal{R}$ includes a possible coupling to the Ricci scalar $\mathcal{R} = d(d+1)$ and \square is the appropriate Laplace operator.

In the following we consider the n -point correlation and vertex functions of the conformally rescaled fields $\phi_a(x) = (-\eta)^{\frac{1-d}{2}} \varphi_a(x)$ in the (interacting) Bunch Davies vacuum state. The latter are conveniently expressed in terms of time ordered products of field operators along a closed contour in (conformal) time; see, e.g., [18]. For instance the two-point function $G_{ab}(x, x') =$

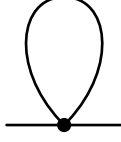


FIG. 1: The self-energy in the limit $N \rightarrow \infty$; see Eq. (10). The internal line corresponds to the propagator G itself, hence the nonperturbative character of this limit.

$\langle T_{\mathcal{C}} \phi_a(x) \phi_b(x') \rangle$, where $T_{\mathcal{C}}$ denotes time-ordering along the contour \mathcal{C} , encodes both the statistical and spectral correlators $F_{ab}(x, x') = \frac{1}{2} \langle \{ \phi_a(x), \phi_b(x') \} \rangle$ and $\rho_{ab}(x, x') = i \langle [\phi_a(x), \phi_b(x')] \rangle$:

$$G_{ab}(x, x') = F_{ab}(x, x') - \frac{i}{2} \text{sign}_{\mathcal{C}}(x^0 - x'^0) \rho_{ab}(x, x'), \quad (3)$$

where the sign function is to be understood on the contour \mathcal{C} . It was shown in [15] that, in the large- N limit, the system only admits $O(N)$ -symmetric solutions. We thus have $\langle \phi_a \rangle = 0$ and $G_{ab} = \delta_{ab} G$.

In the symmetric phase, the four-point correlation and vertex functions $G^{(4)}$ and $\Gamma^{(4)}$ are related by

$$G_{ABCD}^{(4)} = G_{AA'} G_{BB'} G_{CC'} G_{DD'} i \Gamma_{A'B'C'D'}^{(4)} \quad (4)$$

where capital letter indices collectively denote space-time variables and $O(N)$ indices and an appropriate integral/summation over repeated indices is understood. Here, we are interested in computing the equal-time four-point correlator in comoving momentum space $G^{(4)}(\eta, \mathbf{K}_1, \dots, \mathbf{K}_4)$ for superhorizon physical momenta, $-K_i \eta \lesssim 1$. Both the propagator G and the vertex $\Gamma^{(4)}$ have been computed recently in the IR regime in the limit $N \rightarrow \infty$ [15, 19]. Let us briefly review the results relevant for our present purposes.

In comoving momentum space, the two-point function has the free-field-like expression, for $\text{sign}_{\mathcal{C}}(\eta - \eta') = 1$,

$$G(\eta, \eta', K) = \frac{\pi}{4} \sqrt{\eta \eta'} H_{\nu}(-K \eta) H_{\nu}^*(-K \eta') \quad (5)$$

where $H_{\nu}(z)$ is the Hankel function of the first kind and $\nu = \sqrt{d^2/4 - M^2}$. Here, M a self-consistent, dynamically generated mass, to be discussed shortly. In the cases of interest below, $M \ll 1$ and it is convenient to introduce the small parameter $\varepsilon = d/2 - \nu \approx M^2/d$. For superhorizon modes, the statistical and spectral two-point function read

$$F_{\text{IR}}(\eta, \eta', K) = \sqrt{\eta \eta'} \frac{F_{\nu}}{(K^2 \eta \eta')^{\nu}} \quad (6)$$

$$\rho_{\text{IR}}(\eta, \eta', K) = -\sqrt{\eta \eta'} \mathcal{P}_{\nu}^0 \left(\ln \frac{\eta}{\eta'} \right), \quad (7)$$

where $F_{\nu} = [2^{\nu} \Gamma(\nu)]^2 / 4\pi$ and we introduced the function

$$\mathcal{P}_a^b(x) = \frac{\sinh(ax)}{a} e^{-b|x|}. \quad (8)$$

The self-consistent mass M satisfies the gap equation

$$M^2 = m_{\text{dS}}^2 + \sigma \quad (9)$$

where the constant σ is given by the tadpole diagram of Fig. 1. Retaining only the dominant IR contribution in the loop (see [15] for a complete treatment), one gets

$$\sigma = \frac{\lambda}{6N} \langle \varphi^2(x) \rangle \approx \frac{\lambda_{\text{eff}}}{\varepsilon}, \quad (10)$$

where we introduced $\lambda_{\text{eff}} = \lambda F_{\nu} \Omega_d / 12(2\pi)^d$ and $\Omega_d = 2\pi^{d/2} / \Gamma(d/2)$. Equation (9) is solved as

$$M^2 = \frac{m_{\text{dS}}^2}{2} + \sqrt{\frac{(m_{\text{dS}}^2)^2}{4} + d\lambda_{\text{eff}}}. \quad (11)$$

The nonanalytic coupling dependence reflects the non-perturbative IR character of mass generation.

The four-point vertex function can be written as [19]

$$\Gamma_{abcd}^{(4)}(\eta_i, \mathbf{K}_i) = [\eta_1 \dots \eta_4]^{\frac{d-3}{4}} \times \left\{ \delta_{ab} \delta_{cd} \delta_{\mathcal{C}}(\eta_1 - \eta_2) \delta_{\mathcal{C}}(\eta_3 - \eta_4) iD(\eta_1, \eta_3, K_{12}) + \text{perm.} \right\}, \quad (12)$$

where $\delta_{\mathcal{C}}(\eta - \eta')$ is a Dirac delta function on the contour, $K_{ij} = |\mathbf{K}_i + \mathbf{K}_j|$ and 'perm.' denotes the two permutations needed to make $\Gamma^{(4)}$ symmetric. The function D is the two-point correlator of the composite field $\chi \propto \phi^2$:

$$iD(\eta, \eta', K) = -\frac{\lambda}{3N} [\delta_{\mathcal{C}}(\eta - \eta') + iI(\eta, \eta', K)], \quad (13)$$

The first term on the right hand side corresponds, when inserted in Eq. (12), to the tree-level vertex and the function I resums an infinite series of bubble loop diagrams. The one-loop contribution gives a nonlocal power in the IR. Each additional loop leads to an additional power of a large IR logarithm which actually resum to a modified IR power law [19]. The nonlocal function I admits a decomposition in terms of statistical and spectral components as in (3), which read, in the IR,

$$I_F^{\text{IR}}(\eta, \eta', K) = -\frac{\pi_{\rho}}{\sqrt{\eta \eta'}} \frac{F_{\nu}}{(K^2 \eta \eta')^{\bar{\kappa}}}, \quad (14)$$

$$I_{\rho}^{\text{IR}}(\eta, \eta', K) = \frac{\pi_{\rho}}{\sqrt{\eta \eta'}} \mathcal{P}_{\bar{\nu}}^{\varepsilon} \left(\ln \frac{\eta}{\eta'} \right), \quad (15)$$

with $\pi_{\rho} = 2\sigma$, $\bar{\nu} = \sqrt{\nu^2 - \pi_{\rho}}$ and $\bar{\kappa} = \bar{\nu} - \varepsilon$. Here, π_{ρ} is the effective parameter which actually controls the loop expansion. In the following we shall assume $\pi_{\rho} \lesssim 1$ and we introduce the parameter $\bar{\varepsilon} = d/2 - \bar{\nu}$. Expanding the above expression in powers of the effective coupling π_{ρ} generates the whole series of perturbative logarithms. One sees however that the latter breaks down for large time separations $\pi_{\rho} |\ln \eta / \eta'| \gtrsim 1$ and/or deep IR momenta $\pi_{\rho} |\ln K^2 \eta \eta'| \gtrsim 1$.

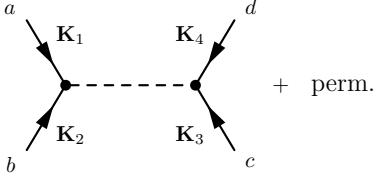


FIG. 2: The equal-time four-point correlator (16). The black lines represent the propagator G and the dashed line represents the nonlocal vertex function (12), which includes the tree-level vertex and an infinite series of bubble loop diagrams.

We now have all the ingredients for our computation of the contribution from superhorizon, IR modes to the four-point equal-time correlator (5). Writing $\int_{\xi} = \int_{\mathcal{C}} d\xi$, the latter can be expressed as the following integral on the time contour \mathcal{C}

$$G_{abcd}^{(4)}(\eta, \mathbf{K}_i) = \delta_{ab}\delta_{cd} \int_{\xi, \xi'} A_{12}(\eta, \xi) B_{12}(\xi, \xi') A_{34}(\xi', \eta) + \text{perm.} \quad (16)$$

where we introduced the functions

$$A_{ij}(\eta, \xi) = G(\eta, \xi, K_i) G(\eta, \xi, K_j), \quad (17)$$

$$B_{ij}(\xi, \xi') = -(\xi\xi')^{\frac{d-3}{2}} D(\xi, \xi', K_{ij}). \quad (18)$$

The tree-level contribution is $\propto \delta_{\mathcal{C}}(\xi - \xi')$ and must be performed separately. It involves the time integral

$$\begin{aligned} & i \int_{\mathcal{C}} d\xi (-\xi)^{d-3} A_{12}(\eta, \xi) A_{34}(\eta, \xi) \\ &= \int_{-\infty}^{\eta} d\xi (-\xi)^{d-3} \{ A_{12}^F(\eta, \xi) A_{34}^P(\eta, \xi) + (12 \leftrightarrow 34) \}, \end{aligned} \quad (19)$$

where we introduced the statistical and spectral components of the function A_{ij} , as in (3), and we used standard manipulations on the contour [24]. Note the symmetry relations $A_{ij}^F(\eta, \xi) = A_{ij}^F(\xi, \eta)$ and $A_{ij}^P(\eta, \xi) = -A_{ij}^P(\xi, \eta)$. Due to the strong IR enhancement of the statistical function (6) as compared to the spectral one (7), we have

$$A_{ij}^F(\eta, \xi) \approx F(\eta, \xi, K_i) F(\eta, \xi, K_j), \quad (20)$$

$$A_{ij}^P(\eta, \xi) = F(\eta, \xi, K_i) \rho(\eta, \xi, K_j) + (i \leftrightarrow j), \quad (21)$$

where we neglected a term $\propto \rho\rho$ in the first line. This is typical of the classical statistical field regime [7, 25] and reveals, in the present context, the classical stochastic nature of dS IR fluctuations [26]. To estimate the contribution from superhorizon modes we replace the integral $\int_{-\infty}^{\eta} d\xi \rightarrow \int_{\eta_0}^{\eta} d\xi$, where η_0 is such that the relevant momenta are superhorizon: $-K_i\eta_0 \lesssim 1$. One can then use the expressions (6) and (7) to compute (19); see below.

The loop contribution in (16) involves the nonlocal

function I in (13). We write

$$\begin{aligned} & \int_{\mathcal{C}} d\xi d\xi' A_{ij}(\eta, \xi) I(\xi, \xi') A_{kl}(\xi', \eta) \\ &= \int_{-\infty}^{\eta} d\xi \int_{-\infty}^{\eta} d\xi' A_{ij}^P(\eta, \xi) I_F(\xi, \xi') A_{kl}^P(\xi', \eta) \\ & - \int_{-\infty}^{\eta} d\xi \int_{\xi}^{\eta} d\xi' A_{ij}^F(\eta, \xi) I_{\rho}(\xi, \xi') A_{kl}^P(\xi', \eta) \\ & - \int_{-\infty}^{\eta} d\xi \int_{\xi}^{\eta} d\xi' A_{kl}^F(\eta, \xi) I_{\rho}(\xi, \xi') A_{ij}^P(\xi', \eta), \end{aligned} \quad (22)$$

replace again $\int_{-\infty}^{\eta} d\xi \rightarrow \int_{\eta_0}^{\eta} d\xi$ and use the IR behaviors (14) and (15). The calculation is straightforward.

Extracting a overall factor and introducing the variable $x = \ln(\eta/\eta_0)$, our final result reads

$$G_{abcd}^{(4)}(\eta, \mathbf{K}_i) = \frac{\lambda}{3N} \frac{F_{\nu}^3}{2\nu} \frac{(-\eta)^{2-4\nu} (-\eta_0)^{2\varepsilon}}{(K_1 \dots K_4)^{2\nu}} \delta_{ab}\delta_{cd} g(x, K_i) + \text{perm.}, \quad (23)$$

with the two momentum structures

$$\begin{aligned} g(x, K_i) &= g_1(x) (K_1^{2\nu} + \dots + K_4^{2\nu}) \\ &+ g_2(x) \frac{(K_1^{2\nu} + K_2^{2\nu})(K_3^{2\nu} + K_4^{2\nu})}{(K_{12})^{2\bar{\kappa}}}. \end{aligned} \quad (24)$$

The function $g_1(x)$ receives contributions from the tree-level vertex and from the last two lines of Eq. (22), while $g_2(x)$ comes from the second line of Eq. (22). We find

$$g_1(x) = \mathcal{L}_{2\varepsilon}(x) + \frac{\pi_{\rho}}{2\nu} \frac{\mathcal{L}_{\varepsilon+\bar{\varepsilon}}(x) - \mathcal{L}_{2\varepsilon}(x)}{\bar{\varepsilon} - \varepsilon}, \quad (25)$$

$$g_2(x) = \frac{\pi_{\rho}}{2\nu} (-\eta_0)^{2\bar{\varepsilon}} \mathcal{L}_{\varepsilon+\bar{\varepsilon}}^2(x), \quad (26)$$

where we defined the function $\mathcal{L}_a(x) = (e^{ax} - 1)/a$. Note the mild dependence on η_0 , which indicates that our assumption of IR dominance is consistent. The first term on the right hand side of Eq. (25) is the tree-level contribution. Loop terms are $\propto \pi_{\rho}$.

Nontrivial IR effects arise in the cases of vanishing, or negative tree-level square mass $m_{\text{dS}}^2 \leq 0$. For $m_{\text{dS}}^2 = 0$, the dynamically generated mass (11) is $M^2 \propto \sqrt{\lambda}$ and

$$\varepsilon = \sqrt{\lambda_{\text{eff}}/d}, \quad \pi_{\rho} = 2d\varepsilon, \quad (27)$$

such that $\bar{\varepsilon} = 3\varepsilon$. We thus get

$$g_1(x) = \mathcal{L}_{2\varepsilon}(x) (1 + \varepsilon \mathcal{L}_{2\varepsilon}(x)), \quad (28)$$

$$g_2(x) = 2\varepsilon (-\eta_0)^{6\varepsilon} \mathcal{L}_{4\varepsilon}^2(x). \quad (29)$$

We see that loop corrections get enhanced by IR effects and are controlled by $\varepsilon \sim \sqrt{\lambda}$. In the regime $\varepsilon|x| \lesssim 1$,

$$g_1(x) \approx x + \varepsilon x^2, \quad g_2(x) \approx 2\varepsilon (-\eta_0)^{6\varepsilon} x^2 \quad (30)$$

and we recover the usual perturbative result [27]: $G^{(4)} \sim \lambda x (1 + \mathcal{O}(\sqrt{\lambda} x^2))$. However, we see that for $\lambda|x| \sim 1$, loop

contributions become of the same order as the tree-level one. In the deep IR regime $\varepsilon|x| \gtrsim 1$, the linear growth in x saturates and one finds the fully nonperturbative result

$$g_1(x) \approx -1/4\varepsilon, \quad g_2(x) \approx (-\eta_0)^{6\varepsilon}/8\varepsilon \quad (31)$$

and $G^{(4)} \sim \sqrt{\lambda}$.

In the case of spontaneous symmetry breaking at tree-level, $m_{\text{dS}}^2 < 0$, IR fluctuations restore the symmetry, resulting in a positive effective square mass $M^2 \propto \lambda$; see Eq. (11). One has, assuming $\lambda \ll |m_{\text{dS}}^2| \lesssim 1$,

$$\varepsilon = \lambda_{\text{eff}}/|m_{\text{dS}}^2|, \quad \pi_\rho = 2|m_{\text{dS}}^2| \quad (32)$$

and thus $\bar{\varepsilon} = \pi_\rho/d \gg \varepsilon$. In this case, we find

$$g_1(x) = \mathcal{L}_\varepsilon(x) \quad \text{and} \quad g_2(x) = \bar{\varepsilon}(-\eta_0)^{2\bar{\varepsilon}} \mathcal{L}_\varepsilon^2(x). \quad (33)$$

We see that loop effects are not suppressed by powers of the coupling. In the deep IR regime $\pi_\rho|x| \gtrsim 1$, one has

$$g_1(x) \approx -1/\bar{\varepsilon}, \quad g_2(x) \approx (-\eta_0)^{2\bar{\varepsilon}}/\bar{\varepsilon}, \quad (34)$$

which exhibits, again, IR enhancement: $G^{(4)} \sim \lambda/|m_{\text{dS}}^2|$.

Finally, we note the specific momentum dependence of the g_2 loop contributions in (23), (24) which is singular whenever the sum of any two momenta approaches zero, $K_{ij} \rightarrow 0$. This loop contribution thus gives a distinct

signature from the tree-level one and, in fact, provides the dominant contribution for such momentum configurations. This is a direct consequence of the IR behavior (14) of the nonlocal four-point vertex (12). In the present case, the latter is given by the two-point function (13) of the operator ϕ^2 which, at large momentum separation is essentially that of a free scalar field of mass $\bar{M}^2 \approx d(\varepsilon + \bar{\varepsilon})$, as noticed in [19]. Loop contributions can thus be seen as describing the exchange of a light (composite) scalar degree of freedom, whereas the tree-level contribution is a contact term. Similar contact and exchange diagrams appear in tree-level calculations of nonGaussian trispectrum for primordial density perturbations in inflationary models and lead to specific nonGaussian signatures [28].

In conclusion, we have obtained an analytic expression of the nonGaussian four-point correlator of an $O(N)$ scalar field in the large- N limit, which shows that dS IR effects can lead to nonperturbative enhancement of loop contributions. We believe our result adds to the understanding of the nontrivial IR physics of light scalar fields in dS space. Although the present model is too simplistic for realistic cosmology applications, our calculation demonstrates how IR effects may spoil the usual perturbative expectations. Possible implications for inflationary cosmology need to be investigated.

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